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ABSTRACT

The purpose of this report is to identify the school setting variables and the teacher competencies which contribute to achievement in the basic skills in mathematics. The search for indicators of competence was accomplished by literature review, expert opinion, research data, and identified student outcomes. The objective was to identify those observable classroom behaviors whose presence or absence is likely to affect student performance. Two types of competency indicators are identified: (1) cognitive-based; and (2) performance-based. Specific instructional competencies are identified and stated in each of the basic skills areas. Five general behaviors were used to group these specific competencies. They are: (1) diagnosis and prescription; (2) organization and management; (3) instruction; (4) growth patterns; and (5) evaluation. (MP)

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INDICATORS FOR LEARNING AND TEACHER COMPETENCIES IN THE BASIC SKILLS

MATHEMATICS

Volume 13 Summer, 1979 Number 1

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RESEARCH AND DEVELOPMENT COUNCIL, INC.

RESEARCH BULLETIN

INDICATORS FOR LEARNING AND TEACHER COMPETENCIES IN THE BASIC SKILLS

MATHEMATICS

by

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TABLE OF CONTENTS

FOREWORD	i
PREFACE	ii
OVERVIEW: INDICATORS OF TEACHER EFFECTIVENESS IN THE BASIC SKILLS	1
INDICATORS RELATED TO LEARNING IN SCHOOL SETTINGS	3
School Characteristics	3
Class Characteristics	5
Program Characteristics	7
Teacher Characteristics	8
Pupil Characteristics	10
Out-of-School Conditions	11
REFERENCES ON INDICATORS	12
MATHEMATICS: TEACHER COMPETENCIES	15
Concepts for the Teaching of Mathematics	16
REFERENCES ON MATHEMATICS	30

FOREWORD

It is difficult to stay on top of current research - whatever your field of endeavor may be. The field of education is no exception, and research on reading, language arts, and mathematics in particular is overwhelming. In this Bulletin, the authors have not only searched out and selected relevant research, but they have organized and interpreted it for the administrator, supervisor, and teacher interacting with students every day.

Drs. Bill Powell and Elroy Bolduc are to be congratulated for their fine work. It is readable, and more important, provides much food for thought. On behalf of FERDC I congratulate them for a job well done.

Bill Breivogel, Guest Editor

PREFACE

Concern for the improvement of basic skills in the State of Florida resulted in the formation of the UF-DOE Basic Skills Project. Financial support for the work was given by the Board of Regents of the State University System through Service Through Application of Research (STAR) program. The State Department of Education provided human resources through committees of consultants. Mrs. Ada P. Puryear, Administrator, Early Childhood and Elementary Education, served as the DOE monitor during the progress of the study.

Evelyn Wenzel's contribution is the section which identifies the teacher competencies in the areas of language arts: listening, speaking, writing, spelling, and handwriting. Elroy J. Bolduc presents the basic teacher competencies for the teaching of mathematics. William R. Powell served as the project director and wrote the portion on reading and the part of the report on indicators of school setting variables. Consultative contributions were made by many of our colleagues on campus. Susan Lubet served as a graduate assistant during part of the study.

Jerri Anne Phipps served as project secretary and typed and retyped the many drafts while the project was in progress and prepared the final report. The interest and effort of all these individuals were definitely appreciated.

William R. Powell

OVERVIEW

INDICATORS OF TEACHER EFFECTIVENESS IN THE BASIC SKILLS

This report has two purposes: (1) to identify the school setting variables (e.g., facilities, materials, equipment, class size, time schedules, etc.) which contribute to achievement in the basic skills; and (2) to identify the teacher competencies which contribute to achievement in the basic skills.

The search for indicators of competence was accomplished by literature review, expert opinion, research data, and identified student outcomes. The objective was to identify those observable classroom behaviors whose presence or absence are likely to affect student performance. Should further evidence support the competencies herein identified, these performances of a teacher can serve as indicators of the effectiveness of that teacher in specific basic skill areas.

In this monograph, two types of competency indicators are identified: (1) cognitive-based criteria; and (2) performance-based criteria. Cognitive-based criteria are the knowledge base, i.e., those facts which must be learned and stored in the memory for use. Performance-based criteria are techniques and processes which must apply and utilize the cognitive items in the context of the classroom. It is conceivable that a teacher could know a set of criteria for instructional placement, yet not know what to do with that criteria in grouping children, making individual assessments, or determining prescription for proper placement. Performance, then, is concerned with the handling of the cognitive systems in the classroom. A teacher cannot teach what she or he does not know, but a teacher can know and not use the knowledge in teaching. The task is to get both systems - cognitive and performance - working together. The consequence of such a fusion of systems will be sustained growth in the basic skills.

In each of the basic skills areas that follow, specific instructional competencies are identified and stated. These specific competencies are grouped, for convenience, by a more general competency category. Five general behaviors were used throughout this report: (1) diagnosis and prescription; (2) organization and management; (3) instruction; (4) growth patterns; and (5) evaluation. Each general category embraces several specific competencies which are listed task by task by the source of knowledge which produces the parallel performance standard.

Tables 1-7 itemize these teacher competencies. These combined indicators, if possessed by the teacher, represent a set of specific competencies considered to characterize an effective teacher of the basic skills.

INDICATORS RELATED TO LEARNING IN SCHOOL SETTINGS

Society has created institutions called schools for developing efficient and effective contexts for learning. Children and adults need to know how to operate in a manner acceptable to others in society. People need to develop specified skills and to know selected information and concepts to perform appropriately in defined social contexts. Schools are primary contributors in that process.

Contexts for learning should be thought of not simply as the physical setting - buildings, space, rooms, surroundings - nor in combination of people (principals, teachers, aides). The settings for learning are constituted by what the pupils are doing and where, when, how, why, and with whom they are doing it. In Figure 1, these different contextual variables and their inter-relationships are illustrated. Each variable provides indicators for pupil performance. The quality of interaction among variables gives strength and vitality to the learning opportunities. The contextual variables which are known to affect learning in the basic skills are the focus here. They are: (1) school characteristics, (2) class characteristics, (3) program characteristics, (4) teacher characteristics, (5) pupil characteristics, and (6) out-of-school conditions. The outcomes are the observed and measured pupil performance in the basic skills - reading, language arts, and mathematics.

Several studies (see references) in the past few years have identified several significant variables affecting learning. The current emphasis on literacy and accountability has contributed to fostering such studies. However, the evidence to date must be viewed as tentative and limited. The instruments presently available for controlled contextual observation as yet are not strong enough for wide and extensive generalizing of the results. There is enough concurrence among studies for some indicators to begin to emerge. It is those items that are presented and discussed in this section of this report, although caution must be maintained in their interpretation.

School Characteristics

Pupil acheivement is higher when the classroom teacher perceives that they have administrative support from the principal and the central office staff, particularly the principal. Support for and leadership in providing opportunities for staff development and provide work-space for academic subjects are positive forces of this leadership.

Other socio-contextual variables affecting the school district are the percent of voting registrants in that district and the average

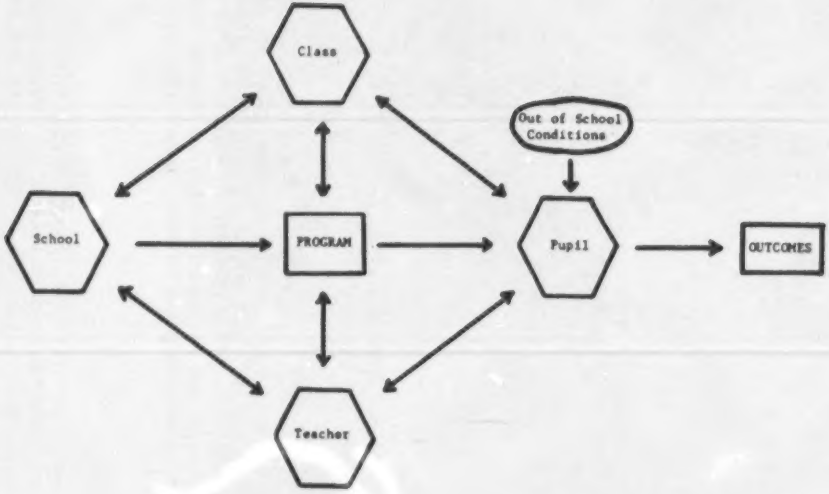

FIGURE 1. Contextual Variables for Basic Skill Achievement

income per family unit in the district where the school resides. Both of these items have overtones of literacy attainment, interest in societal affairs, and socio-economic well being. Table 1 lists these indicators which facilitate higher achievement at the school level.

TABLE 1. School Indicators

Characteristics
Perceived administrative leadership and support
Staff development and in-service activity
Percent of space used for academics
Percent of voting registrants in school district
Average income of school district

The factor of size (community, school district, or school population) is neither a positive nor a negative force on basic skill achievement. The type of physical facilities (old, remodeled, new) apparently also has a neutral effect on pupil outcome as does the type of organizational patterns within the school. Size, facilities, and organizational strategies, in and of themselves, are not prime contributors to measured pupil performance.

However, the density of district population (urbanness), the number of special programs in the school, and the percent of black membership in both the school and community are indicators of schools with lowered pupil achievement. Socio-economic factors and society's attempt to off-set those factors, i.e., special programs are created and funded for the handicapped, compensatory, etc. are reflected in these findings.

Class Characteristics

Higher pupil achievement at the class level within a school is associated with the quality of the classroom atmosphere, i.e., a sense

of order and purpose, positive relationships, and pleasure in learning. The level of classroom control maintained by the teacher is a contributing factor. Surprisingly, perhaps to some, higher achievement, in several studies, is associated with less grouping within the class limits. However, this latter indicator may be a proxy variable for direct instruction time. The greater the teacher's time is spread across several groups, the less time is available by the teacher with any particular group. However, less grouping does not mean no grouping. What may be suggested in the data is that in the schools studied there may have been too many groups used in recent attempts to individualize instruction. Grouping is directly related to classroom control factors and direct instructional time from the teacher.

The presence of minority groups present in the classroom is a positive influence on outcomes. In school systems where there are numerous schools of varying size, the evidence would suggest that black youngsters achieve better in the smaller schools with smaller classes. Table 2 provides the factors positively related to better achievement at the classroom level.

TABLE 2. Class Indicators

Characteristics
Classroom atmosphere
Level of classroom control
Grouping not overextended within the class
Minority groups present
Black youngsters in smaller school environments

The size of a given class shows ambiguous results in achievement. However, a class size of thirty-three or less pupils has indicated a positive trend towards higher achievement patterns. Classroom enrollment in the studies reviewed may have been fairly uniform in size and, if so, then class size would not have shown positive relationships.

Achievement grouping shows an unclear pattern of relationships to pupil achievement. This finding is consistent with other studies through the years. Ability grouping or power grouping, per se, is not clearly and consistently associated with higher pupil performance. One factor, however, has shown a negative relationship to achievement - that o. pupil transfer rate. Evidently, some stability in a class setting has its contributing qualities.

Program Characteristics

A program with an emphasis on the basic skills brings the corresponding result of higher achievement in those areas. It should, however, be a balanced total program not limited solely to basic skill subjects. Programs with clear objectives with emphasis on cognitive development that are designed with concern for scope and sequence are clearly associated with higher pupil performance.

TABLE 3. Program Indicators

Characteristics
Balanced curriculum program
Basic skill emphasis
Cognitive development emphasized
Stated objectives of desired behavior
A scope and sequence in the basic skill areas
Adequate material and equipment support
Total reading activity
More silent reading than oral
A system of instruction
Teacher training in the system

The necessary materials and equipment for program operation are necessary. There is some hint in the literature that too much material can be a detractor as well as too little resources. Overstimulation can be a distractor in learning.

In reading instruction, the data suggest that the total reading activities, not just phonics and other learning to read components, is a significant factor in reading achievement. Also, the amount of silent reading time is positively related to higher performance. Conversely, oral reading activities, if extensive, is not a facilitating force in obtaining higher achievement in reading.

What is important in basic skill achievement is that some system of instruction is clearly specified and followed. Further, teachers need to have training in <u>that</u> system - not generally, but specifically. The implication of such a finding is obvious: while colleges educate teachers for <u>any</u> system of instruction, each employing school system must additionally train the teacher in the specifics for their particular program.

The impact of pre-school education programs on basic skills achievement is not yet apparent. This is probably due to the fact that such programs are relatively new and time for their contribution has not to date made its impact. Nevertheless the trend line is positive in this regard.

<u>Teacher Characteristics</u>

The most recent finding about indicators of teacher competencies is that effective teaching practices will differ by subject matter and grade level taught. These findings mean that teachers use different techniques, as well as content, in different subjects like reading and mathematics. Not only do the techniques and content vary, but so does the grade level. Second grade teachers will have different instructional strategies than fifth or sixth grade teachers. The implication for the preparation of teachers is clear that the objectives for training primary and intermediate grade teachers may need to be different.

Interaction patterns between the teacher and pupils follow a similar pattern as subject and grade level. The presentation pattern, the type and frequency of teacher questions, the teacher's reaction to pupil responses, the nature of feedback to pupils, and the degree of teacher circulation are positive influence on pupil performance. The pattern of these techniques will vary when the grade level and subject matter varies.

Teacher characteristics such as teacher enthusiasm, realistically high expectations, teacher effort, and teacher morale are indicators of high achievement. Basically, a teacher who has a desire to teach and does, gets results. Other external teacher factors are experience and certification. Certification is likely to reflect the number of years of training, but the evidence would suggest that where that training is obtained makes a difference. Teachers from rated college programs produce better results. The salary level of the school staff, principals and teachers, have long been a positive indicator of teacher performance.

TABLE 4. Teacher Indicators

Subject matter taught
Grade level taught
Interactive strategies (discussing, questions, feedback, etc.)
Teacher enthusiasm
High expectations
Teacher effort
Teacher morale
Experience
Certification
Rate college training
Salaries of teachers and principles

Factors such as the percent of married teachers, the percent of black teachers and the use of teacher aides are apparently negligible.

However, if the teacher aide qualified as certified personnel, then that indicator is significantly related to pupil achievement.

Pupil Characteristics

Direct instructional time is a highly significant indicator identified in the recent literature. Instructional time and student achievement are positively related. The amount of time allocated to a skill area appears to be a contributor to the amount learned. A distinction, however, needs to be made between allocated time and time on task (engaged time). It is time on task that is the critical factor.

Engaged time has a quality of student attention. The student must be involved with active attention in the task at hand. The task at hand may be interacting either with the teacher or the learning materials. The teacher directs the process. Whether a student is in a group or doing seatwork is not the basic criterion. What is important is whether the teaching practice directs, guides, and engages the pupil.

The evaluation of pupil progress is an indicator of pupil performance. Teachers, to provide direct instruction, need diagnostic evaluation data, process evaluation information, and product evaluation data. What the pupils need to have is feedback from these evaluations - the sooner, the better.

TABLE 5. Pupil Indicators

Characteristics
Direct instructional time
Time on skill area
Time on task
Evaluation of pupil progress with feedback
Attendance
Female

Females do better in the basic skill subjects than males. This is an expected pattern. Attendance is a positive influence on achievement, although it may be a reflection of allocated learning time. In essence, what the research on pupil characteristics shows is that direct engaged learning time with reinforcement signals is significantly related to pupil achievement. In general, more time yields more learning.

Out-of-School Conditions

The parent's educational level is the best single indicator of a covert socio-economic variable which influences pupil performance. Other factors which show a positive relationship to achievement are the parent's income, occupation, and aspiration level for their children. The home situation, such as, the number living at the home, when the adults are home, the language used in the home, etc., have also facilitating influences on a pupil's achievement.

TABLE 6. Out-of-School Indicators

Characteristics
Parent's educational level of attainment
Parent's occupation
Parent's income
Parent aspiration level for child
Home situation (language, number in home space, when adults are home, etc.)

From the preceding discussion, it is clear that several interacting socio-contextual variables are significantly related to pupil performance in the basic skills. No one set of characteristics alone are great enough to predict pupil achievement. It takes a combination of forces. What a child brings to school and what the school brings to the child are all part of the complete context for learning.

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MATHEMATICS: TEACHER COMPETENCIES

The development of a set of teacher competencies to enable a teacher to teach basic skills cannot be based directly on research studies. (There have been few mathematics studies investigating any specified lists of teacher competencies and their relationship with the acquisition of basic skills in mathematics by students.) The list of teacher competencies developed in this paper is based on the basic skills list developed in an earlier phase of this project "Specifying Basic Skills" and draws heavily on several reports of national groups such as the National Council of Teachers of Mathematics and reports of conferences on basic skills such as the "Conference of Basic Mathematical Skills and Learning" sponsored by the U. S. Department of Health, Education, and Welfare.

There is disagreement within the mathematics community about any list of basic skills, a list of teacher competencies also generates the same disagreement. The list of competencies in this paper, and the skills presented in "Specifying Basic Skills" do follow the position prevalent within the mathematics community in that it views basic mathematical skills as including much more than computation. As stated in the Report of the Conference on Basic Mathematical Skills and Learning, there is a popular notion that mathematics is arithmetic, and that the most basic mathematical skill, perhaps the only one, is fluency with written computational algorithms (22). The competencies listed here include areas other than computation ability, the various areas are interrelated and may overlap with each other. The teacher skills listed are basic to an ability to help students to reason effectively in varied situations.

A list of several major reports on basic skills and teacher competencies follows. These reports have had a major influence on the mathematics community in the United States and were strongly in the preparation of this report.

Bell, Max S. "What Does Everyman Really Need from School Mathematics?" The Mathematics Teacher, 67:196-202, 1974.

Edwards, E. L., Eugene D. Nichols, and Glyn H. Sharpe. "Mathematical Competencies and Skills Essential for Enlightened Citizens", The Mathematics Teacher, 65:671-677, 1972.

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The teacher competencies are presented in two categories - Cognitive (the teacher knows) and Performance (the teacher can do...). The order in which they are listed does not presuppose any notion of an hierarchy and should not be interpreted as any indication of a priority ordering. As stated by NCTM there may be teachers who are highly successful, but their abilities and training may result in a combining of competencies ways that prevent a one-to-one correspondence with the competencies listed (42). This phenomena makes it very difficult to use competencies for evaluation purposes. There is always the problem and danger of minimum lists becoming norms. The list presented here is a minimal one. It is esstential that teachers know significantly more than they are expected to teach.

Concepts for the Teaching of Mathematics

Growth Patterns. Any efficient teaching patterns must somehow be correlated with the nature of the learning process. There have been many different theories of learning - each having different implications for teaching. Though no single theory of learning has supplied a total explanation of human learning, there are some agreements among educational psychologists. One such apparent agreement is the fact that learners progress through various stages of growth. Teachers should be familiar with stages of development of cognitive processes and be able to recognize factors which affect their development.

The work of Piaget and others has many implications for the teaching and learning of basic skills. At least one study found a relationship between Piaget-type class-inclusion tasks and ability of students to perform missing-addend problems (32). Other studies have found a relationship between the stage developments outlined by Piaget and achievement on tests of mathematical ability (5,46). Perhaps the most important single proposition that the teacher can derive from Piaget's work is that children, especially young ones, learn best from concrete activities (24).

Stages of learning also exist within topics. For efficient learning, an exploratory phase should proceed the phase of verbalization and concept formation, and eventually the material learned should be merged in, and contribute to the integral mental attitude of the learner (51). A common misinterpretation of Piaget's four stages of cognitive growth is that concrete materials are not needed by adolescent pupils (2). At all levels, including adolescence, the pupil will be far more capable of "doing" and "understanding" in an action mode than expressing himself in a verbal mode (47). With this misinterpretation, teachers are often tempted to present material in a formal mode far too early.

Diagnosis and Prescription. Competencies needed by teachers for diagnosing learning difficulties in mathematics were suggested as long ago as 1935 by Bruechner, these include:

1. Teachers must have a clear conception of the function & objectives of mathematics.

2. Teachers be able to use effective means to determine thought processes that would ordinarily be left unanalyzed.

Early work in diagnosis (1920's) was mostly studies of frequency of errors in computation. Diagnosis has recently made great advances and has even become a sub-specialty within the field of mathematics education. The best sources of data of learning problems are probably careful observations by the teacher and analysis of written work. Roberts (55) has developed procedures for determining problems in computation and Ashlock (7) has developed an extensive list of common computational error patterns and some ways of correcting them.

For many teachers, their knowledge of mathematics and diagnostic skills are too deficient to recognize the patterning of errors that reveal the nature of student misunderstanding (30). Children's errors are not all or low "mathematical aptitude." Errors regularly have a systematic basis - some are the result of a correct procedure incorrectly applied. Children's mistakes seem to derive from strategies of one kind of another (27). Too often lack of versatility in mathematics causes teachers to miss opportunities to provide just the right experience that might pierce the barrier to a student's comprehension.

Skills. The subset of competencies under the heading "skills" is the largest of the various subjects. For this paper "skills" refers to the basic mathematical content needed by the teacher and methodological

techniques for presentation of these skills to students. The competencies listed here closely follow the guidelines set forth by the National Council of Teachers of Mathematics (42) and the National Council of Supervisors of Mathematics (43).

A current major problem related to students' acquisition of mathematical skills is that of the mathematical competence of many elementary school teachers (10). As stated by the National Council of Mathematics a teacher of mathematics at any level should know and understand mathematics substantially beyond that which he may be expected to teach (42). There is much evidence indicating that many elementary school teachers are not well enough prepared to teach all the mathematics expected of them. One study showed that nearly 25% of Elementary Education majors scored below the 20th percentile on a standardized test and less than 5% scored above the 90th percentile. Using scores for 8th and 9th grade students as a norm, 55% of these prospective teachers scored below the median. (53)

The remainder of this section will be divided into sub-sections related to various content areas.

Numeration. A thorough understanding of numeration systems is essential in order to apply basic arithmetic facts to higher order numbers. A childs experience with absolute quantities is limited to around 100 and less. This is only minimally acceptable as a context for teaching addition and subtraction and unacceptable for teaching multiplication and division. The place value concepts allow grouping to aid in expanding the student's experience with larger quantities (13).

The use of the number line affords a concrete representation to show relationships between numbers. As mentioned earlier one possible reason for difficulty in students learning of mathematical skills is that little time is given to making the concepts meaningful through the use of physical materials. There seems to be a rush to get to symbols and abstractions. Students taught only in a symbolic mode in primary grades develop inaccurate concepts of mathematics (35). Primary teachers in particular should have a background in developmental theory with emphasis placed on the application of this theory to the classroom (35).

Sets. Sets have been associated with "new math" and a controversial topic within elementary mathematics. The use of set-concepts - not set-theory - to teach and explain addition, multiplication, cardinality curves, etc., has proved a useful technique. One study noted that "operations on sets are more meaningful to the student than operations on numbers," since sets are concrete objects (61). The data from a second study suggest that numerals are learned only after counting operations for sets of the size represented by the

numerals are well established (66).

Computation. As content gets beyond the whole number stage studies show that teachers still have a tremendous amount of trouble with decimal and percentage type problems. At least 59% of teachers in one study could not answer, "An increase from $16 to $20 is _____% increase." (21) This study did show that the "average" 1973 teacher was significantly more accurate in computational skills than his 1930 counterpart -- but there was no significant change in either the top 27% or the bottom 27%.

The standard algorithms for the basic operations form a large part of basic skills. Pupils cannot process information requiring combinations of large quantities until they have mastered the algorithms for computation (13). There are several algorithms for each operation and teachers should be familiar with these as each one contains some advantages. For example one investigation compared the conventional division algorithm (distributive algorithm) with the subtractive algorithm and found several advantages for each one:

1. Subtract method was especially effective for children of low ability.

2. Children taught subtractive method had a better understanding of the process of division.

3. Children taught the conventional (distributive) method achieved higher problem solving scores. (64)

Teachers also need to recognize that problem situations require students to view operations in several ways. Subtraction, for example is normally taught in a "take away" mode where problem situations may require a student to consider subtraction from an "additive", or "comparative" mode. (26, 58) Similarly division can be considered from either a "measurement" or a "portition" situation (29, 68). At least one study has shown that use of two algorithms was not too difficult for students and required no more than only one algorithm.

Teachers must also recognize non-standard algorithms. Standard algorithms represent only part of a child's skills. The child also develops "invented" procedures to perform calculations. (27) One problem in children's acquisition of basic skills is the teachers use of mathematical neuristics that do not reflect the child's way of viewing things.

Teachers should also be familiar with the so-called "low stress algorithms" for the four basic operations, these seem to have

some merit though research results have not been released. These algorithms are characterized by consise, definable, easily read, supplementary notation to record every step (33).

Geometry. Geometry is the study of space, form, and relationships. There are two aspects of geometry which must be considered by teachers, metric geometry which deals with the assignment of numbers to geometric objects, e.g. calculation of volumes, areas, lengths, etc. and nonmetric geometry, the identification of special relationships and of geometric patterns (57).

The National Council of Mathematics suggests that teachers should be able to (1) extract concepts of two- and three-dimensional geometry from the real world of the child; (2) discuss properties of simple geometric figures such as line, line segment, angle, triangle quadrilateral, circle, perpendicular and parallel lines, pyramid, cube, sphere; and to (3) determine one-, two-, and three-dimensional measures of common figures (42). (This rather complete competency was suggested for teachers of Early Childhood and Primary Grades.)

Geometry, per se, is probably not basic to any set of minimal human needs...but the parts of metric geometry closely related to measurement systems are essential skills. (13) The National Council of Supervisors of Mathematics state "As a minimum skill, students should be able to measure distance, weight, time, capacity and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools." (43)

Piaget has suggested that children's psychological development is "inverted" in relation to the development of geometrics over time. (48) Topological relations were the last to be developed by mathematicians but are the most basic relations. The child appears to reproduce these stages first and then goes to Euclidian structures (37). Teachers should have geometry and measurement experiences from an exploration and performing level. Merserve states that "prospective elementary school teachers have a serious need for experiences involving explorations in geometry in the pedagogical spirit that they should use in their own teaching." (39)

Measurement. Teachers should learn to think in the metric system and not through conversion problems (31). Since the units in the metric system are related by powers of ten the teacher must not only consider and provide readiness experiences for measuring in metric but also consider whether children can understand decimal fractions (60).

In an introductory stage for measurement students should first become familiar with the properties of the objects to be measured...atributes, and then learn that measurement is the assignment of a number of this attribute. From this viewpoint teachers should have knowledge of and experience with arbitrary and non-standardized units in order to teach measurement concepts and the need for measuring (41).

Graphing. We live in a data oriented society. Graphing techniques are necessary in order to be able to condense numerical information into more manageable and/or meaningful terms (43). Even though numerical information is encountered regularly, few people have the training to organize such information or to examine it critically or use it effectively (41). The teacher must be able to choose appropriate methods to display information and be able to generate meaningful problems using such data.

Number Theory and Statistics. Statistics as a formal topic is probably not a necessary skill for human interaction. It is important that the teacher be aware of elementary notions of statistical techniques to show how mathematics is used to help to make predictions (election forcasts, tv ratings, etc.). They should learn to identify situations where immediate past experience does not affect the likelihood of future events (43).

The interpretation of numerical information is needed for proper evaluation of every day matters (such as advertising claims) and is indispensable for the solution of policy questions (41).

Field Axioms. The field axioms, like set theory, may have contributed to the disillusionment with "new math". If a formal introduction is made too early the study of axioms becomes a meaningless act. The concepts themselves, however, are used over and over in mathematics. Each new mathematics process is a repeated use of basic principles rather than being an entirely new set of rules (15).

Within mathematics itself, the ability to follow an argument is surely basic. Teachers should know how to develop and refine this sense in the student (11).

Problem Solving. "Learning to solve problems is the principle reason for studying mathematics" (43). Unfortunately "application" in school mathematics usually means "word problem" and for most of these, the main task of the student is the translation of technical jargon of mathematical prose into simpler language and then into suitable symbolic form. These usually apply to few realistic situations (41).

Exposure to well conceived concrete applications serves two purposes, it permits pupils to use skills already developed and motivates them toward new and deeper mathematical insights (57).

Estimation is a very important skill in mathematics. The results from "National Assessment" show that this skill is not well developed in students. Only 54% of 17 year old students gave an acceptable response on one item requiring estimation and only 40% on a second item (16).

Teachers also need to learn to establish a problem solving atmosphere for children. In this type of situation the role of the teacher changes from one of providing all the answers to using skillful questioning to draw answers from students (35).

There are approaches teachers should know from problem solving:

1. Making a generalization through pattern finding.
2. Using successive approximations (trial and error).
3. Decomposing a problem into simpler subproblems.
4. Simplifying a problem.
5. Recalling a similar problem.

It is necessary that teachers be familiar with such strategies and develop problems which use such strategies for their solution. On some occasion real problems require only simple recall of some facts and use of an algorithm - more often real problems the situation is more complicated - several processes or a strategy must be used to solve the nonroutine applications of mathematics (38).

Organization and Management. Barriers to learning mathematics are not likely to be reduced by programs alone. Just as important as mathematics competency are the skills of program management. Teachers need to develop competency working with small groups and individuals. Needless boredom and frustration could be avoided with better management skills (30).

Students need more instruction using physical objects and manipulatives. If teachers are trained using these manipulatives, they are more likely to use them in their classroom. One place where manipulatives can be used is in the mathematics laboratory. The mathematics laboratory means many different things - it is sometimes thought of as an "approach to learning mathematics" and sometimes as

"a place where students can be involved in learning mathematics." (54) Research results are far from conclusive however they do indicate that laboratory approaches can be used practically and effectively and appear to be useful methods for a teacher (65).

One important management task is to balance drill and practice and concept development.

Several studies investigated the ratio of time spend on developmental activities compared to drill and practice. The evidence suggests that arithmetic skills are better learned by spending more time on developmental tasks (teacher demonstration, explanations, group-discussions, laboratory activities, etc.) and less time on drill and practice (exercises from textbooks, kits, dittos, etc.). (40, 59, 67) One study on use of classtime in arithmetic discovered that 76% of his time used for review of homework and oral or written drill and only 23% on developmental activities. Another report suggested, "In most cases an increased amount of exploration time results in a better understanding of the topic, better selection, and thus less need for drill." (55)

Evaluation. Any systematic attempt to develop basic skills must include a teachers ability to evaluate and report pupil progress. Evaluation is essential to measure progress and to plan program improvement. The competencies listed in this section include more than an ability to interpret standardized tests. Teachers must also have knowledge of criterion reference tests (competency tests and norm-referenced tests. In criterion-referenced tests items are generated to measure specidif objectives of a program and establish a students level of mastery. The possibility of directly relating a test to a set of objectives gives this type of test a distinct advantage in individual student evaluation (41). Norm-referenced tests provide a score which will compare a student's performance with norming population.

Teachers must learn to observe, interview and assess skills by manipulative tasks in areas which paper and pencil tests do not measure adequately.

Observation of students' feelings towards mathematics are another important part of the evaluation task. Attitudes and interests are affective things having to do with feelings. Attitudes are thought to exert a dynamic, directive, influence and may be related to the teaching and learning of mathematics. Teachers should know several scales used to measure attitudes and interests even though there is no consistant body of research to support the popular belief that

there is a significant positive relationship between pupil attitudes towards mathematics and pupil achievement in mathematics (62).

One study reported evidence that the teacher's attitude is significantly related to the student's attitude and achievement (49). Another study concluded that the inhibition produced by "anxiety" appears to overwhelm any motivating effects. In general it appears that anxiety and mathematics are related - high anxiety associated with lower achievement (9).

Several studies have investigated the influence of teachers attitudes towards mathematics upon pupil attitudes and achievement.

One study reported a significant relationship between teacher attitude and pupil preference for mathematics for students with a high IQ (>110) but no such significance with pupils of lower IQ. Others have reported agreement between teacher and student preference for mathematics while Abrego found no such relationship. (17, 34)

Calculators and computers have not been included in the list of competencies for teachers. The very cheap "4-function" calculator will have definite impact on the curriculum and basic skills in the very near future. Teachers will have to be trained in the use of hand calculators and in computers and these areas will have to be added to lists of basic skills and teacher competencies.

TABLE 7. MATHEMATICS: TEACHER COMPETENCIES

Know (Cognitive)	Can Do (Performance)
GROWTH PATTERNS	
1. Know stages of development of cognitive processes of children as they relate to mathematics and factors which affect their development Example: The four levels described by Piaget i Sensori-motor ii Pre-operational iii Concrete Operations iv Formal Operations	1. - Select appropriate strategies and materials for use in classroom - Assess "conservation" levels of students - Prepare lessons using concrete, pictorial, and symbolic materials - Adapt instruction in accordance with individual differences
DIAGNOSIS AND PRESCRIPTION	
2. How to distinguish among types of mathematical errors e.g. careless errors conceptual errors	2. - Use techniques to locate errors in thinking, as well as in computation, e.g. have student "think aloud"
3. How to analyze work to determine types of errors made	3. - Plan appropriate learning experiences based on error analysis
SKILLS	
4. Understanding of decimal numeration system to read and write large and small numbers, and to determine relationships between numbers	4. - Explain base ten system by using grouping procedures and expanded notation - Create and use real world examples for both large and small numbers - Use models such as the number line to show relationships among numbers

TABLE 7. MATHEMATICS: TEACHER COMPETENCIES cont.

Know (Cognitive)	Can Do (Performance)
SKILLS cont.	
5. Basic concepts of set theory as they relate to elementary mathematics	5. - Use set theory concepts to teach and explain concepts such as - cardinality - equivalence - addition - multiplication, etc. - Use set theory concepts to teach and define geometric concepts such as curves, lines, angles, etc.
6. The four basic operations (+, -, x, ÷) with - whole numbers - rational numbers in fraction notation in decimal notation, including percents	6. - Develop and use models for teaching operations with numbers (e.g. number line) - Explain standard algorithm for basic operations - Recognize and test new or non-standard algorithms - Use appropriate situations for the several types of percent problems - Use various types of problem situations to develop mathematical understanding, e.g. "Take away," "Additive," and "Comparison" subtraction problems
7. Basic concepts of geometry	7. - Use appropriate models to teach two and three dimensional concepts - Teach intuitive topological concepts such as closed curves, interior and exterior, etc. - Develop intuitive concepts of congruent and similar figures, area, volume, etc.

TABLE 7. MATHEMATICS: TEACHER COMPETENCIES cont.

Know (Cognitive)	Can Do (Performance)
SKILLS cont.	
	7. cont. - Develop and use formulas for area, perimeter, and volume of geometric figures - Teach use of basic tools for simple figure drawing and measurements
8. Basic concepts of measurement and understanding of metric system	8. - Use arbitrary units of measurement to teach measurement concepts and the realization of a need for measure - Show relationship between metric and American systems and use approximate conversions between systems if conversions are appropriate - Use school, home and community resources in measurement activities
9. Understand basic graphing techniques	9. - Show value of various graphing techniques (e.g. bargraphs, circle graphs, pictographs, etc.) - Use real world data to develop realistic problems - Help student chools appropriate graphing techniques to display information
10. Understand basic concepts of number theory and statistics	10. - Develop concepts such as LCM, GCD, primes, etc. - Develop simple probability activities, e.g. as a problem situation involving fractions - Develop problem solving situations for study of concepts such as mean, median, and mode

TABLE 7. MATHEMATICS: TEACHER COMPETENCIES cont.

Know (Cognitive)	Can Do (Performance)
SKILLS cont.	
11. Understand the Twelve Field Axioms as related to elementary mathematics	11. - Use properties such as commutative and associative to assist in teaching basic facts - Develop distributive property by use of physical models - Use field axioms (concepts) to develop algorithms - Show "proof" of elementary statements at an appropriate level of sophistication
12. Create and solve real world problems	12. - Develop problems which relate to child's experiences - Develop problem lessons to teach estimation of answers and "rounding off" numbers - Teach systematic approaches to problem solving - Using examples, teach students to recognize what data is relevant in a problem
ORGANIZATION AND MANAGEMENT	
13. Know scope and sequence of Topics in mathematics in elementary grades	13. - Introduce topics at proper level of sophistication for ability of students - Use the concept of a spiral curriculum to assist in determining proper achievement levels in a topic
14. Understand rationale of laboratory activities in mathematics	14. - Use game and puzzle situations to develop mathematical concepts - Use "science experiments" in mathematical settings

TABLE 7. MATHEMATICS: TEACHER COMPETENCIES cont.

Know (Cognitive)	Can Do (Performance)
ORGANIZATION AND MANAGEMENT cont.	
	14. cont. - Use methods which convey understanding of concepts and facts not just rote memorization - Prepare lessons and units which create a balance between concept formation and drill and practice
15. Know several instructional strategies for presenting topics	15. - Prepare and teach lessons using a wide variety of activities and materials - Structure "small task" instruction so student can focus on more immediately attainable goals - Use inductive learning activities to teach students and emphasize inductive reasoning
EVALUATION	
16. Know role of evaluation in classroom a. extension of learning b. diagnostic aid	16. - Construct pre-tests and post-tests for topics in elementary curriculum - Use observation as well as written measures for evaluation - Observe attitudinal measures as well as achievement and aptitude
17. Understand and interpret standardized and criterion referenced test data as well as teacher made data	17. - Determine questions which measure a skill or concept - Develop classroom "tests" to measure particular characteristics - Describe acceptable responses to assessment tasks - Use results of student evaluation to improve instruction

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